Integration of reaction rate formula

Given

$$\frac{\Delta[A]}{\Delta t} = -k \cdot [A]^n$$

To find the concentration at any point in time t, first find the differential

$$\lim_{\Delta t \to 0} \frac{\Delta[A]}{\Delta t} = \frac{d[A]}{dt}$$

Rearrange to integrate with respect to the concentration

$$\frac{d[A]}{dt} = -k \cdot [A]^n \;\therefore\; dt = \frac{1}{-k \cdot [A]^n} d[A]$$

Integrate with the range from initial to final concentrations over period t

$$t - t_0 = \int_{[A]_0}^{[A]} \frac{1}{-k \cdot [A]^n} d[A]$$

Extract the constant, multiply both sides by $-k$, and rewrite the order and change in time for the sake of ease

$$-k \cdot \Delta t = \int_{[A]_0}^{[A]} [A]^{-n} d[A]$$

Solve the integral using the power rule and rearrange

$$-k \cdot \Delta t = \left(\frac{[A]^{-n+1}}{-n+1} - \frac{{[A]_0}^{-n+1}}{-n+1} \right), n \neq 1$$

$$\frac{[A]^{-n+1}}{-n+1} = -k \cdot \Delta t + \frac{{[A]_0}^{-n+1}}{-n+1}, n \neq 1$$

In the case of the first order, the exponent becomes -1 and must be integrated differently

$$-k \cdot \Delta t = \int_{[A]_0}^{[A]} [A]^{-1} d[A]$$

Integrate using the rule for power of -1 and rearrange

$$-k \cdot \Delta t = \ln|[A]| - \ln|[A]_0|$$

$$\ln|[A]| - k \cdot \Delta t + \ln|[A]_0|$$

Integrated reaction laws for 0, 2, and higher: (Notice a negative is factored in to reverse signs.)

Zeroth order $\quad [A] = -k \cdot \Delta t + [A]_0$

Notice in the following that -1 is factored out.

Second order $\quad \frac{1}{[A]} = k \cdot \Delta t + \frac{1}{[A]_0}$

Third order $\quad \frac{1}{2 \cdot [A]^2} = k \cdot \Delta t + \frac{1}{2 \cdot {[A]_0}^2}$

Fourth order $\quad \frac{1}{3 \cdot [A]^3} = k \cdot \Delta t + \frac{1}{3 \cdot {[A]_0}^3}$

Fifth order $\quad \frac{1}{4 \cdot [A]^4} = k \cdot \Delta t + \frac{1}{4 \cdot {[A]_0}^4}$